Eur. Phys. J. C 31, 245–260 (2003)
Digital Object Identifier (DOI) 10.1140/epjc/s2003-01330-2

THE EUROPEAN PHYSICAL JOURNAL C

Sparticle masses in 4D product-group gauge theories

N. Maru[1,a,*], K. Yoshioka[2,b]

[1] Theoretical Physics Laboratory, RIKEN (The Institute of Physical and Chemical Research), 2-1 Hirosawa, Wako, Saitama 351-0198, Japan
[2] Department of Physics, Kyushu University, Fukuoka 812-8581, Japan

Received: 6 June 2003 /
Published online: 2 October 2003 –

Abstract. In this paper, we investigate the possibility to have supersymmetry breaking with background modulus fields in four-dimensional product-group gauge theories. The vacuum expectation values of the modulus fields satisfy several relations, and their dependences of the action can be fixed by loop-level consistency of the model. We examine the mass spectrum of vector and matter multiplets up to one-loop order of perturbation theory. As an application, it is found that the properties of higher-dimensional supersymmetry breaking are well captured in the various limits in the moduli space. In particular, we have finite radiative corrections to the Higgs masses in the case that is shown to be equivalent to the boundary condition breaking of supersymmetry.

1 Introduction

Motivated by the gauge hierarchy problem, physics with extra dimensions has provided new insights on various aspects of particle physics, cosmology and astrophysics [1, 2]. There, however, seems to be some difficulty in discussing the issues in the ultraviolet regime since higher-dimensional theories are generally non-renormalizable in the usual sense. Recently, a four-dimensional framework describing the physical nature of higher-dimensional theories has been proposed in [3,4]. This framework consists of a gauge theory of product group and scalar fields in the bi-fundamental representations of the nearest-neighbor gauge groups. If one assumes that the bi-fundamental scalars develop vacuum expectation values (VEVs), the gauge groups are broken to a simple diagonal subgroup. It has been shown that with a sufficiently large number of gauge groups, the mass spectrum of gauge fields is equivalent in the intermediate energy regime to the Kaluza–Klein (KK) mass spectrum of five-dimensional gauge theory. This approach is providing new tools for four-dimensional model building and moreover for understanding unexplored properties of higher-dimensional theories. In fact, various applications along this line have been discussed in the literature [5].

In [6], we have studied a model with supersymmetry (SUSY) breaking induced by modulus fields which are naturally incorporated into the above framework. The modulus fields are found to satisfy some relations and to have non-trivial dependences of the action, if one wants to see a description of higher-dimensional effects. In [6], we have identified the modulus form of the action and verified it for a five-dimensional vector multiplet by explicitly calculating the tree-level mass spectrum of the gaugino and the associated adjoint scalar. Various types of SUSY-breaking mass spectra predicted in higher-dimensional models appear as the corresponding limits in the parameter space of the modulus F terms.

In this paper, as an extension of the previous results, we formulate a four-dimensional model with general matter multiplets, and evaluate one-loop radiative corrections to the mass spectrum. We first fix the modulus couplings to the matter multiplets as well as to the vector ones, and then calculate radiative corrections to the gaugino and scalar soft masses in detail. It will be found that the above-mentioned resemblances of the mass spectrum to the existing higher-dimensional models still hold even at the quantum level. In addition, we study the ultraviolet behavior of the radiative corrections and find in some case the correction to the Higgs masses to be finite. Such a case is explicitly shown to be closely related to the boundary condition breaking of supersymmetry.

This paper is organized as follows. In Sect. 2, we first review the action for vector multiplets and the resulting tree-level mass spectrum of the gauginos and adjoint scalars. We also fix the modulus couplings to the matter multiplets and calculate the (SUSY-breaking) masses at the classical level. With the complete action at hand, radiative corrections to the masses of vector and matter multiplets are studied in detail in Sect. 3. Section 4 gives some arguments about the finiteness property of loop corrections found in Sect. 3. There we will particularly pay attention to some

[a] e-mail: maru@postman.riken.go.jp
[b] e-mail: yoshioka@higgs.phys.kyushu-u.ac.jp
[*] Special Postdoctoral Researcher

relation between the finite corrections and the boundary condition breaking of supersymmetry. Section 5 is devoted to a summary of the paper.

2 Classical action

2.1 Vector multiplets

The model we consider is a four-dimensional SUSY gauge theory with the gauge groups $G_1 \times G_2 \times \ldots \times G_N$. We simply assume all the gauge couplings g_i to have the universal value g. The four-dimensional $N = 1$ vector multiplet V_i of the G_i gauge theory contains a gauge field A^i_μ and a gaugino λ^i. In addition, we have the $N = 1$ matter multiplets Q_i $(i = 1, \ldots, N)$ in the bi-fundamental representation; that is, Q_i transforms as $(\square, \bar{\square})$ under the (G_i, G_{i+1}) gauge groups. (G_{N+1} is identified with G_1.) The field content of the model is then summarized below:

	G_1	G_2	G_3	$\ldots$	G_N
Q_1	$\square$	$\bar{\square}$	1	$\ldots$	1
Q_2	1	$\square$	$\bar{\square}$	$\ldots$	1
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
Q_N	$\bar{\square}$	1	1	$\ldots$	$\square$

(2.1)

The gauge-invariant Lagrangian for the vector multiplets and the Q_i is written as

$$\mathcal{L}_{\mathrm{vec}} = \sum_i \left[\int \mathrm{d}^2\theta\, S W_i^\alpha W_{\alpha i} + \mathrm{h.c.} + \int \mathrm{d}^2\theta \mathrm{d}^2\bar{\theta}\, K_Q(S, S^\dagger)\, Q_i^\dagger \mathrm{e}^{\sum V} Q_i \right], \qquad (2.2)$$

where W_i^α is the field strength superfield of the G_i gauge group, and S is a dilaton-like background superfield whose scalar component determines the value of g [see (2.6)]. In fact, one may introduce a dilaton field for each gauge theory G_i. We now take the universal value of the gauge couplings and use S as a representative of such dilaton fields. The normalization function $K_Q(S, S^\dagger)$ of the matter field Q_i is given by [6]

$$K_Q(S, S^\dagger) = \frac{8}{1/S + 1/S^\dagger}, \qquad (2.3)$$

which can be fixed by several classical-level consistencies as will be discussed at the end of this section. Moreover, this form of K_Q will be found in the next section to be important for the loop-level consistency of the model.

If one considers the case that the Q_i have VEVs proportional to the unit matrix, $\langle Q_i \rangle = v$,[1] the gauge symmetries are broken down to its diagonal subgroup G. With this symmetry breaking, the mass eigenvalues and eigenstates of the gauge fields are found to be

$$m_n = 2v \sin\frac{n\pi}{N}, \qquad \widetilde{A}^n_\mu = \frac{1}{\sqrt{N}} \sum_{j=1}^N (\omega_n)^j A^j_\mu, \qquad (n = 0, \ldots, N-1), \qquad (2.4)$$

where $\omega_n = \mathrm{e}^{2\pi i n/N}$. One can see the KK tower of massive gauge fields as well as the massless one which belongs to the unbroken subgroup G. It is interesting to note that the spectrum matches that of five-dimensional gauge theory with compact extra dimensions [3, 4]. Actually, in the large N limit with v/N fixed, the gauge boson masses approximately become

$$m_n \simeq \frac{2vn\pi}{N} = \frac{n}{R}, \qquad (2.5)$$

where the compactification radius R is identified as $2\pi R = N/v$. With this identification, v is interpreted (for fixed N) as an ultraviolet cutoff of the effective four-dimensional theory, and N is the number of KK modes which exist between the cutoff and compactification scales.

In the absence of SUSY breaking, the gauginos and the associated adjoint scalars, which are linear combinations of the original fields λ_i and Q_i, have mass spectra degenerate with that of the gauge bosons (2.4). Before examining SUSY-breaking mass splitting, it may be instructive to recall the modulus fields in our model and their mutual relations. In addition to the S introduced above, there is a modulus Q which determines the size of the universal VEV v. The modulus Q may be a normalized composite field or a linear combination of the Q_i. Throughout this paper, we assume just for simplicity that the moduli forms of the dilatons S_i and the Q_i are invariant under the "translation" transverse to the four dimensions, that is, they are i independent. The forms of the VEVs of these moduli are then defined by

$$S = \frac{1}{4g^2} + F_S\theta^2, \qquad Q = v + F_Q\theta^2. \qquad (2.6)$$

Furthermore, it will turn out to be useful to define the additional background modulus fields with the following VEVs:

$$S_4 = \frac{1}{4g_4^2} + F_{S4}\theta^2, \qquad S_5 = \frac{1}{4g_5^2} + F_{S5}\theta^2, \qquad T = \frac{1}{R} + F_T\theta^2, \qquad (2.7)$$

where g_4 and g_5 are the effective gauge coupling constants in four and five dimensions, respectively. It is important to notice that all these modulus fields are not independent variables. By comparing the Kaluza–Klein theory with the low-energy description of the model below the cutoff v, one finds the following matching conditions between the parameters [3, 4]:

$$2\pi R = \frac{N}{v}, \qquad g_4^2 = \frac{g^2}{N}. \qquad (2.8)$$

[1] More generic cases with non-universal values of the gauge couplings and VEVs of the Q correspond to gauge theories on curved backgrounds [7, 8] and low-energy gauge symmetry breaking [9]. We do not consider these possibilities in the present work

Table 1. The modulus F terms of the typical SUSY-breaking scenarios

	F_S	F_Q	F_{S4}	F_{S5}	F_T
dilaton ($F_T \equiv 0$)	F_S	0	NF_S	vF_S	0
moduli ($F_{S4} \equiv 0$)	0	F_Q	0	$\frac{1}{4g^2}F_Q$	$\frac{2\pi}{N}F_Q$
radion ($F_{S5} \equiv 0$)	$\frac{-F_Q}{4g^2 v}$	F_Q	$\frac{-NF_Q}{4g^2 v}$	0	$\frac{2\pi}{N}F_Q$

The first condition in (2.8) was already adopted in (2.5), which was required to match the spectrum with that of KK theory. The second condition is regarded as a volume suppression of bulk gauge coupling in compactifying an extra dimension. In addition to these, the normalization of the gauge kinetic terms provides a relation between g_4 and g_5, irrespectively of how to define the five-dimensional model,

$$g_5^2 = 2\pi R g_4^2. \qquad (2.9)$$

These three relations among the couplings suggest the following relations among the modulus fields:[2]

$$S_4 = NS, \qquad S_5 = QS, \qquad T = \frac{2\pi}{N}Q. \qquad (2.10)$$

We thus find that the modulus fields S_4, S_5 and T are expressed in terms of the two moduli S and Q. As a result, the two-dimensional parameter space of the F components of S and Q describes SUSY-breaking patterns in our model. In [6], we clarified that several limits in this parameter space describe bulk SUSY-breaking patterns which have been discussed in the literature.[3] In Table 1, the typical cases are presented for the dilaton/moduli dominated SUSY-breaking [11] and for SUSY breaking by the radius modulus F term [12,13]. The specification of each limit is also given in the table. For example, the dilaton dominance scenario is defined by $F_T = 0$, which is in turn translated to the limit $F_Q = 0$ in our model [see (2.10)]. In the following, we will study the SUSY-breaking effects from these modulus fields and examine the sparticle spectrum of the model.

If one introduces suitable potentials for the modulus fields, their VEVs are fixed to some point in the parameter space. For example, since S is a dilaton for each gauge group, stabilization mechanisms proposed in the literature could be incorporated in the present model. The situation is similar for the modulus Q. Moreover in properly describing five-dimensional theory, Q is assumed to be stabilized by relevant potential terms [3,10]. However, details of potential form are not relevant to us. Without referring to specific models, we explore the whole parameter space of the modulus F terms and then focus on several limits corresponding to bulk SUSY-breaking patterns. We do not try to construct the specific dynamics for the modulus fields where potential couplings are tuned for realizing the fifth dimension. Our aim here is not to present five-dimensional theories. It is only the relevant region of moduli space where our model reproduces bulk SUSY-breaking scenarios. In other words, the present framework contains unexplored four-dimensional phenomena of SUSY breaking.

Now the SUSY-breaking mass spectrum can be derived from the Lagrangian (2.2) with turning on the modulus F terms. The result is written by use of the above-defined moduli and their relations,

$$m_{\lambda_{n\pm}} = \pm\sqrt{m_n^2 + \left|\frac{F_{S5}}{2\langle S_5\rangle}\right|^2} - \frac{F_T}{2\langle T\rangle}, \qquad (2.11)$$

$$m_{c_n}^2 = m_n^2 + 2\,\mathrm{Re}\left(\frac{F_{S5}^*}{\langle S_5\rangle}\frac{F_T}{\langle T\rangle}\right), \qquad (2.12)$$

where $m_{\lambda_{n\pm}}$ and m_{c_n} are the KK masses of gauge fermions and adjoint scalars. The bracket $\langle\ \rangle$ denotes a VEV of its lowest scalar component. The positive (negative) sign in $m_{\lambda_{n\pm}}$ corresponds to the gaugino (the Goldstone fermion) masses in the supersymmetric limit. Note that the results are expressed by five-dimensional quantities only. In particular, it is found that the zero-mode gaugino mass is given by *both* the radius modulus and the five-dimensional dilaton. This result is expected from (2.9), which implies that S_4 depends both on T and S_5 ($TS_4 = 2\pi S_5$).

We here explicitly show the several limits in order. As mentioned before, the dilaton-dominated SUSY breaking is characterized by the condition $F_T = 0$. In this limit, we find

$$m_{\lambda_{n\pm}} = \pm\sqrt{m_n^2 + |2g^2 F_S|^2}, \quad m_{c_n}^2 = m_n^2 \qquad \text{[dilaton]}. \qquad (2.13)$$

The gaugino mass spectrum is indeed the one expected in supergravity models. All the KK states including the zero mode receive the universal SUSY-breaking contribution from the modulus S, the two level-n spinors are degenerate in mass, and the mass splittings between bosons and fermions are equal for all KK modes. These results may be understood from the fact that the dilaton field commonly couples to any field in the theory.

On the other hand, the limit of the moduli-dominated SUSY breaking is defined by $F_{S_4} = 0$ and then leads to

$$m_{\lambda_{n\pm}} = \pm\sqrt{m_n^2 + \left|\frac{F_Q}{2v}\right|^2} - \frac{F_Q}{2v}, \qquad m_{c_n}^2 = m_n^2 + 2\left|\frac{F_Q}{v}\right|^2 \qquad \text{[moduli]}. \qquad (2.14)$$

It is interesting to note that even when the SUSY-breaking effect is turned on, the zero-mode gaugino remains mass-

[2] The 1PI and holomorphic gauge couplings differ only at higher-loop level in perturbation theory

[3] Supersymmetry breaking that is local in the higher-dimensional bulk was also studied within four-dimensional product-group gauge theories [10]

less.[4] This is exactly the tree-level spectrum predicted in this class of SUSY-breaking models [11,14]. Since $F_{S4} = 0$ by definition, the zero-mode gaugino mass is vanishing and is shifted at loop level by string threshold corrections or effects of bulk fields. The situation may be similar to the model where the vector multiplets behave as messengers of SUSY breaking and sparticle soft masses are calculated from the wave function renormalization in four dimensions [15]. There may also be an intuitive explanation for the above type of spectrum. That is, a non-zero F term of the modulus which determines the KK masses does not induce tree-level SUSY-breaking masses for zero modes. This is because these two mass terms are proportional to the KK numbers. In our case, such a modulus corresponds to the one whose scalar component obtains a VEV $\propto 1/R$ and is given by $Q \propto T$.

The last example, $F_{S_5} = 0$, realizes the SUSY breaking with the radius modulus field. The sparticle mass spectrum in this limit becomes

$$m_{\lambda_{n\pm}} = \pm m_n - \frac{F_T}{2\langle T\rangle}, \quad m_{c_n}^2 = m_n^2 \qquad \text{[radion]}. \quad (2.15)$$

The gaugino mass matches the one calculated in [13]. The vanishing SUSY-breaking masses of the adjoint scalars agree with the fact that this limit is equivalent to the Scherk–Schwarz mechanism [16], which is now applied to the $SU(2)_{\mathrm{R}}$ symmetry under which the adjoint scalar is a singlet and hence does not get a symmetry breaking mass.

Before closing this subsection, we comment on the normalization function $K_Q(S, S^\dagger)$. The form of K_Q (2.3) is determined so that it satisfies several non-trivial requirements. First, the holomorphy requires the normalization of Q_i to be $\langle K_Q\rangle = 1/g^2$, which leads to the same normalization for the vector and adjoint chiral multiplets of the low-energy G gauge theory. Moreover, the radius superfield in our model becomes independent of the dilaton superfield [see (2.10)]. This seems plausible since an undesirable relation between the theta angle and the graviphoton field does not arise. Another consistency concerns the 5-5 component of the five-dimensional metric g_{55}. In a continuum five-dimensional theory, the kinetic terms of the bosonic fields along the fifth dimension have the dependence of $g_{55}\sqrt{g_{55}}\,g^{55} \propto 1/R$. In the present model, the second term in the Lagrangian (2.2) becomes the kinetic energy in the continuum limit, and its modulus dependence is given by $\langle K_Q(S, S^\dagger)\, Q^\dagger Q\rangle$. Equation (2.3) then indicates $\langle K_Q Q^2\rangle \sim \langle S Q^2\rangle \sim \langle S_5 T\rangle$. Consequently, the metric dependence agrees for a fixed value of the five-dimensional coupling g_5.

2.2 Matter multiplets

Next let us discuss matter multiplets, which can be regarded as hypermultiplets if one takes the five-dimensional limit. As in the case of vector multiplets, we will determine the proper form of the moduli dependences of the matter action and then examine the SUSY-breaking mass spectrum at tree level. One-loop corrections to the masses of the scalar components in these matter multiplets will be studied in Sect. 3.

In addition to the fields in the previous subsection, we introduce a set of vector-like chiral multiplets Φ_i and $\bar{\Phi}_i$ for each gauge group G_i:

$$\begin{array}{c|cccccc} & G_1 & G_2 & G_3 & \dots & G_N \\ \hline \Phi_1 & \square & 1 & \dots & \dots & 1 \\ \bar{\Phi}_1 & \bar{\square} & 1 & \dots & \dots & 1 \\ \Phi_2 & 1 & \square & \dots & \dots & 1 \\ \bar{\Phi}_2 & 1 & \bar{\square} & \dots & \dots & 1 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \Phi_N & 1 & 1 & \dots & \dots & \square \\ \bar{\Phi}_N & 1 & 1 & \dots & \dots & \bar{\square} \end{array} \qquad (2.16)$$

The gauge-invariant Lagrangian for the matter sector can be described by

$$\mathcal{L}_{\mathrm{mat}} = \sum_{i=1}^{N}\left[\int \mathrm{d}^2\theta \mathrm{d}^2\bar{\theta}\, K_h(S, S^\dagger)\left[\Phi_i^\dagger \mathrm{e}^{2V_i}\Phi_i + \bar{\Phi}_i \mathrm{e}^{-2V_i}\bar{\Phi}_i^\dagger\right] + \int \mathrm{d}^2\theta\left[Y\bar{\Phi}_i Q_i \Phi_{i+1} + Z\bar{\Phi}_i\Phi_i\right] + \text{h.c.}\right], \quad (2.17)$$

where Φ_{N+1} is identified to Φ_1. We have assumed that the coupling constants are universal for simplicity. The background chiral superfields Y and Z in the superpotential are the modulus fields providing the Yukawa and mass parameters. As will be seen below, these fields are expressed in terms of the moduli S and Q. Similar to the vector multiplet case, it is a non-trivial problem to fix the moduli dependence of the normalization function $K_h(S, S^\dagger)$. That issue will be discussed later.

2.2.1 Superpotential

First we study the moduli dependence of the superpotential terms. It is convenient to rescale the matter multiplets as $(\Phi, \bar{\Phi}) \to \langle K_h\rangle^{-\frac{1}{2}}(\Phi, \bar{\Phi})$ so that the kinetic terms are canonical. In the rescaled basis, the superpotential contribution to the supersymmetric mass terms reads

$$W_{\mathrm{mass}} = \frac{1}{\langle K_h\rangle}\sum_{i,j}\bar{\Phi}_i\Big(\langle Y\rangle\, v\,\delta_{i+1,j} + \langle Z\rangle\,\delta_{i,j}\Big)\Phi_j. \quad (2.18)$$

It is easily found that the relation $\langle Y\rangle = \sqrt{2}\,\langle K_h\rangle$ must be satisfied if one requires the matter spectrum to be equivalent to that of a vector multiplet (i.e. that of KK theory in the five-dimensional limit). This implies the moduli form

$$Y = K_h(S, S). \quad (2.19)$$

[4] The level $n = 0$ spinor being affected by the non-zero F terms is the Goldstone fermion associated to gauge symmetry breaking

Given this relation, diagonalizing the supersymmetric mass term leads to

$$W_{\text{mass}} = m_n \tilde{\bar{\Phi}}_m \delta_{mn} \tilde{\Phi}_n + \big(\langle Z \rangle / \langle K_h \rangle + v \big) \tilde{\bar{\Phi}}_n \tilde{\Phi}_n, \quad (2.20)$$

where the supersymmetric eigenstates $\tilde{\Phi}_n$ and $\tilde{\bar{\Phi}}_n$ $(m, n = 0, \ldots, N-1)$ are defined similar to those of the vector fields (2.4). The irrelevant phase factors have been absorbed in the field redefinitions so that the mass eigenvalues are real. In the absence of non-zero F terms, the mass eigenstates take the same form as the gauge fields. The first term in the right-handed side of (2.20) corresponds to the KK masses and the second one to a bare mass parameter of the matter multiplet. When the bare mass m is defined by $\langle Z \rangle / \langle K_h \rangle + v = m/R$, the modulus Z should take the form

$$Z = \left(\frac{2\pi m}{N} - 1 \right) Q K_h(S, S). \quad (2.21)$$

Now that the moduli dependences of the superpotential part are fully determined, their contribution to the SUSY-breaking masses can be analyzed. From (2.17), (2.19) and (2.21), we find the mass eigenvalues of the spinor components:

$$m_{\psi_n} = m_n + \frac{m}{R} \qquad (n = 0, \ldots, N-1). \quad (2.22)$$

The scalar soft mass terms from the superpotential are written

$$\mathcal{L}_{\text{W}} = \quad (2.23)$$
$$\left[\left\langle \frac{\partial \ln K_h(S,S)}{\partial \ln S} \right\rangle \frac{F_S}{\langle S \rangle} + \frac{F_T}{\langle T \rangle} \right] \tilde{\bar{\phi}}_m \left(m_n + \frac{m}{R} \right) \delta_{mn} \tilde{\phi}_n$$
$$+\text{h.c.},$$

where ϕ and $\bar{\phi}$ are the scalar components of Φ and $\bar{\Phi}$, respectively. This is the holomorphic mixing mass term of ϕ and $\bar{\phi}$. In the rescaled basis, the mixing masses depend on the Kähler factor K_h, but they should be cancelled if one includes the full contribution of the modulus fields as will be seen below.

2.2.2 Kähler potential

Scalar masses also come from the Kähler potential since it has the moduli dependence and so is non-canonical. The masses of the spinor components (2.22) do not get changed by the Kähler terms. For the rescaled fields defined above, we read off the Lagrangian (2.17),

$$\mathcal{L}_{\text{K}}$$
$$= \sum_{i=1}^{N} \left[|F_{\phi_i}|^2 + \left\langle \frac{\partial \ln K_h}{\partial S} \right\rangle F_S F^\dagger_{\phi_i} \phi_i + \left\langle \frac{\partial \ln K_h}{\partial S^\dagger} \right\rangle F^\dagger_S \phi^\dagger_i F_{\phi_i} \right.$$
$$+ \left\langle \frac{1}{K_h} \frac{\partial^2 K_h}{\partial S \partial S^\dagger} \right\rangle |F_S|^2 \phi^\dagger_i \phi_i + |F_{\bar{\phi}_i}|^2 + \left\langle \frac{\partial \ln K_h}{\partial S} \right\rangle F_S \bar{\phi}_i F^\dagger_{\bar{\phi}_i}$$
$$\left. + \left\langle \frac{\partial \ln K_h}{\partial S^\dagger} \right\rangle F^\dagger_S F_{\bar{\phi}_i} \bar{\phi}^\dagger_i + \left\langle \frac{1}{K_h} \frac{\partial^2 K_h}{\partial S \partial S^\dagger} \right\rangle |F_S|^2 \bar{\phi}_i \bar{\phi}^\dagger_i \right]. \quad (2.24)$$

Integrating out the matter auxiliary components F_{ϕ_i} and $F_{\bar{\phi}_i}$ and moving to the supersymmetric mass basis, we have

$$\mathcal{L}_{\text{K}} =$$
$$- \sum_{n=0}^{N-1} \left[\left(|m_{\psi_n}|^2 + \left\langle \frac{\partial \ln K_h}{\partial S} \frac{\partial \ln K_h}{\partial S^\dagger} - \frac{1}{K_h} \frac{\partial^2 K_h}{\partial S \partial S^\dagger} \right\rangle F_S F^\dagger_S \right) \right.$$
$$\times \left(\tilde{\phi}^\dagger_n \tilde{\phi}_n + \tilde{\bar{\phi}}_n \tilde{\bar{\phi}}^\dagger_n \right)$$
$$\left. + 2 m_{\psi_n} \left\langle \frac{\partial \ln K_h}{\partial S} \right\rangle F_S \, \tilde{\bar{\phi}}_n \tilde{\phi}_n + \text{h.c.} \right]. \quad (2.25)$$

Combining this with the superpotential contribution to the scalar masses (2.23) and noting that $\langle \partial \ln K_h(S,S)/\partial S \rangle = 2 \langle \partial \ln K_h / \partial S \rangle = 2 \langle \partial \ln K_h / \partial S^\dagger \rangle$, we finally obtain the scalar mass matrix for matter multiplets:

$$\mathcal{L}_{\text{scalar}} =$$
$$- \sum_{n=0}^{N-1} \begin{pmatrix} \tilde{\phi}^\dagger_n \\ \tilde{\bar{\phi}}_n \end{pmatrix} \quad (2.26)$$
$$\times \begin{pmatrix} |m_{\psi_n}|^2 + \varGamma \left| \frac{F_S}{\langle S \rangle} \right|^2 & -m^*_{\psi_n} \frac{F^*_T}{\langle T \rangle} \\ -m_{\psi_n} \frac{F_T}{\langle T \rangle} & |m_{\psi_n}|^2 + \varGamma \left| \frac{F_S}{\langle S \rangle} \right|^2 \end{pmatrix} \begin{pmatrix} \tilde{\phi}_n \\ \tilde{\bar{\phi}}^\dagger_n \end{pmatrix},$$

$$\varGamma \equiv \left\langle \frac{\partial^2 \ln K_h^{-1}}{\partial \ln S \partial \ln S^\dagger} \right\rangle. \quad (2.27)$$

Thus the tree-level mass eigenvalues are given by

$$m^2_{\phi_{n\pm}} = |m_{\psi_n}|^2 \pm m_{\psi_n} \frac{F_T}{\langle T \rangle} + \varGamma \left| \frac{F_S}{\langle S \rangle} \right|^2$$
$$(n = 0, \ldots, N-1). \quad (2.28)$$

One can see in this formula that the effect of the modulus S is controlled by the factor $\varGamma$, which is a function of the normalization constant in the matter Kähler term, while the F_T part does not depend on it. For example, $\varGamma = b/4$ for the Kähler form

$$K_h = (S + S^\dagger)^b X(S) X(S^\dagger), \quad (2.29)$$

where b is a constant and X is an arbitrary function. We find that $b = 1$ and $X =$ constant are the appropriate form if one wants to describe five-dimensional theory. This choice is supported by examining the tree-level spectrum and radiative corrections, which will be discussed in the following sections. Such a Kähler function indeed satisfies the following non-trivial consistencies:

(i) the mass spectrum of scalar matters including SUSY-breaking effects coincides with that of gauginos,

(ii) the cutoff dependences of radiative corrections become consistent with known results, and
(iii) the moduli dependence of the action has the proper form similar to the argument given at the end of Sect. 2.1.

2.2.3 Various limits

We have determined all the moduli dependences of the action except for K_h and presented the superparticle spectrum for vector and matter multiplets in four-dimensions: (2.4), (2.11), (2.12), (2.22), and (2.28). It has been shown that for the gaugino mass, the several limits of the F terms suggest the five-dimensional properties of SUSY breaking. Let us examine the scalar mass spectrum for the typical cases shown in Table 1. The first is the dilaton dominance limit defined by $F_T = 0$. It is found from (2.28) that the scalar mass eigenvalues in this limit become

$$m_{\phi_{n\pm}}^2 = |m_{\psi_n}|^2 + \Gamma \left| \frac{F_S}{\langle S \rangle} \right|^2 \qquad \text{[dilaton]}. \tag{2.30}$$

As expected, the SUSY-breaking contributions are universal for all the scalar fields. If one takes the Kähler form (2.29) with $b = 1$, the spectrum (2.30) (with a vanishing bare mass m) is the same as that of the gauge fermions (2.13). The second limit we consider is the moduli dominance which leads to the mass eigenvalues

$$m_{\phi_{n\pm}}^2 = |m_{\psi_n}|^2 \pm m_{\psi_n} \frac{F_Q}{v} \qquad \text{[moduli]}. \tag{2.31}$$

It is interesting to note that the spectrum is predicted independently of the detailed form of the matter Kähler function K_h. Moreover, the zero mode ($n = 0$) does not get SUSY-breaking effects and remains massless. These features are certainly shared with gauginos. In the approximation that SUSY-breaking effect is much smaller than the compactification scale, the mass eigenvalues of the excited modes are written $m_{\phi_{n\pm}} \simeq m_{\psi_n} \pm F_Q/2v$, which is also consistent with the KK gauginos.

In the limit $F_{S5} = 0$, we have

$$m_{\phi_{n\pm}}^2 = |m_{\psi_n}|^2 \pm m_{\psi_n} \frac{F_T}{\langle T \rangle} + \Gamma \left| \frac{F_T}{\langle T \rangle} \right|^2 \qquad \text{[radion]}. \tag{2.32}$$

The mass eigenvalues (2.32) have the same form as those of the gauginos when $b = 1$ for the matter Kähler form. Note that the masses of the excited modes approximately agree with those in the moduli dominance limit. The only difference is whether the zero mode is massless or not, which mode is identified with the low-energy degree of freedom in the five-dimensional viewpoint.

We thus find from the tree-level analysis of the mass spectrum that $b = 1$ in the Kähler form (2.29) is a suitable choice for the normalization of matter multiplets. The remaining functional dependence X will be fixed by loop-level consistency of theory.

2.3 Orbifolding

The analysis above has been performed for the case that corresponds to the circle compactification of the fifth dimension. It is straightforward to extend it to the compactification on the line segment. For vector multiplets, what we have to do is removing a bi-fundamental field, e.g. Q_N. This procedure leads to a four-dimensional vector multiplet as the light degrees of freedom. The gauge anomaly arising from removing Q_N can be supplemented by introducing appropriate chiral fields charged under the G_1 and G_N gauge symmetries. The effects of these “local” fields can be neglected in the large N limit. For matter multiplets, the absence of one anti-chiral multiplet, e.g. $\bar{\Phi}_N$, leaves a chiral zero mode of the fundamental representation of the diagonal subgroup G. If one chiral multiplet, e.g. Φ_1, is removed, the zero mode is the anti-fundamental representation. In these cases, suitable anomaly cancellations are also required. The gauge-invariant Lagrangian and its moduli dependences are almost the same as before. For example, when $\bar{\Phi}_N$ is removed, the mass eigenvalues and eigenstates are given by

$$m_n = m_{\psi_n} = 2v \sin\left(\frac{n\pi}{2N}\right) \qquad (n = 0, \ldots, N-1), \tag{2.33}$$

$$\widetilde{\Phi}_n = \sqrt{\frac{2}{2^{\delta_{n0}} N}} \sum_{j=1}^{N} \cos\left(\frac{2j-1}{2N} n\pi\right) \Phi_j,$$

$$\widetilde{\bar{\Phi}}_n = \sqrt{\frac{2}{N}} \sum_{j=1}^{N-1} \sin\left(\frac{j}{N} n\pi\right) \bar{\Phi}_j. \tag{2.34}$$

This result can be obtained for the case of removing Φ_1 by exchanging $\Phi \leftrightarrow \bar{\Phi}$, and in a similar way for the vector multiplets. The SUSY-breaking mass formulas do not change except for the expression of m_n. It is, however, noted, in contrast to the previous section, that the bare mass parameters of matter multiplets can be introduced only if there is a set of matter multiplets which leaves vector-like massless modes.

3 Quantum analysis

We have discussed SUSY-breaking effects through the moduli S and Q, and we calculated the tree-level spectrum by determining the proper moduli dependences of the action. We particularly showed that on the several specific lines in the parameter space of F_S and F_Q, the sparticle spectra are consistent with the high-dimensional SUSY-breaking patterns. Note that, in some cases, the low-energy degrees of freedom do not obtain SUSY-breaking effects and remain massless in the tree-level approximation. It is then important, e.g. for realistic model building, to include radiative corrections. In this section, we will calculate various types of one-loop corrections to gaugino and Higgs scalar masses from gauge and Yukawa interactions. In the following, the analysis will be performed in the case of orbifold compactification.

3.1 Gaugino masses

3.1.1 Vector contribution

The first type of one-loop corrections which contribute to zero-mode gaugino masses involves vector and adjoint scalar multiplets running in the loops. Let us recall that the mass matrix of the nth massive gauge fermions takes the form

$$-\frac{1}{2}\left(\tilde{\lambda}_n\ \tilde{\chi}_n\right)\begin{pmatrix}\frac{F_S}{2\langle S\rangle} & m_n \\ m_n & -\frac{F_Q}{\langle Q\rangle}-\frac{F_S}{2\langle S\rangle}\end{pmatrix}\begin{pmatrix}\tilde{\lambda}_n \\ \tilde{\chi}_n\end{pmatrix}+\text{h.c.}, \quad (3.1)$$

where χ_n are the Goldstone spinors associated with the gauge symmetry breaking. The eigenvalues $m_{\lambda_{n\pm}}$ of this matrix were given by (2.11), and the mixing angle between λ_n and χ_n upon diagonalization satisfies

$$\tan 2\theta_n = \frac{2m_n\langle S_5\rangle}{F_{S5}}. \quad (3.2)$$

At the component level, there are two types of one-loop diagrams for the gaugino two-point function, which come from gauge fields and adjoint scalars, respectively,

$$I_{\text{gauge}} = \left(\frac{g}{\sqrt{N}}\right)^2 C_2(G) \times \sum_{n=0}^{N-1}\int\frac{\mathrm{d}^4k}{\mathrm{i}(2\pi)^4}\frac{g_{\mu\nu}}{(p-k)^2-m_n^2}\sigma^\mu \times\left[\frac{\cos^2\theta_n}{m_{\lambda_{n+}}-\not{k}}+\frac{\sin^2\theta_n}{m_{\lambda_{n-}}-\not{k}}\right]\sigma^\nu, \quad (3.3)$$

$$I_{\text{adj}} = \left(\frac{\sqrt{2}g}{\sqrt{N}}\right)^2 C_2(G) \quad (3.4)$$
$$\times\sum_{n=0}^{N-1}\int\frac{\mathrm{d}^4k}{\mathrm{i}(2\pi)^4}\frac{1}{m_{c_n}^2-(p-k)^2}\left[\frac{\sin^2\theta_n}{m_{\lambda_{n+}}-\not{k}}+\frac{\cos^2\theta_n}{m_{\lambda_{n-}}-\not{k}}\right],$$

where p is the four-momentum of the external gaugino line and $C_2(G)$ is the quadratic Casimir operator for the adjoint representation of the gauge group G. With the introduction of an ultraviolet cutoff Λ, the divergence parts are calculated to be

$$I_{\text{gauge}}^{\text{div}} = \frac{C_2(G)}{16\pi^2}\left(\frac{g}{\sqrt{N}}\right)^2 \quad (3.5)$$
$$\times\sum_{n=0}^{N-1}\left(\not{p}-4\left(m_{\lambda_{n+}}\cos^2\theta_n+m_{\lambda_{n-}}\sin^2\theta_n\right)\right)\ln\Lambda^2,$$

$$I_{\text{adj}}^{\text{div}} = \frac{C_2(G)}{16\pi^2}\left(\frac{g}{\sqrt{N}}\right)^2 \quad (3.6)$$
$$\times\sum_{n=0}^{N-1}\left(\not{p}+2\left(m_{\lambda_{n+}}\sin^2\theta_n+m_{\lambda_{n-}}\cos^2\theta_n\right)\right)\ln\Lambda^2.$$

These divergences should be renormalized by appropriate counter-terms. Note in particular that the total one-loop divergent correction to the gaugino mass is

$$\delta m^{(1)} = \frac{2NC_2(G)}{16\pi^2}\left(\frac{g}{\sqrt{N}}\right)^2\frac{F_{S5}}{\langle S_5\rangle}\ln\Lambda^2. \quad (3.7)$$

It is interesting that, since this total mass divergence is proportional to F_{S5}, there appears no ultraviolet divergence in the limit of the radius modulus SUSY breaking. This is indeed a consistent result and will be discussed in detail later on. On the other hand, the divergences generally appear in other cases – for example, in the dilaton/moduli dominant cases.

The finite part of the gauge field correction is given by

$$I_{\text{gauge}}^{\text{fin}} = \frac{-2NC_2(G)}{16\pi^2}\left(\frac{g}{\sqrt{N}}\right)^2\frac{F_T}{\langle T\rangle}. \quad (3.8)$$

We have used the approximation that the zero-mode gaugino mass, i.e. the SUSY-breaking mass scale is much smaller than the KK masses. The finite correction from I_{adj} can also be estimated in a similar way and is found to be $-\frac{1}{2}I_{\text{gauge}}^{\text{fin}}$. We thus obtain the zero-mode gaugino mass up to one-loop level,

$$m_{\lambda_{0\pm}} = \pm\frac{F_{S5}}{2\langle S_5\rangle}-\frac{F_T}{2\langle T\rangle}-\frac{2NC_2(G)g_4^2}{16\pi^2}\frac{F_T}{\langle T\rangle}. \quad (3.9)$$

The four-dimensional effective gauge coupling g_4 is defined in (2.8). One can find two important points from this expression for the one-loop gaugino masses. Firstly, the radiative correction is proportional to F_T but not to F_S. This fact seems to agree with a result of string-inspired supergravity models [11]. There, one-loop corrections to the gaugino masses arise from string threshold corrections or the effects of bulk fields and are specified by the modulus field T. The second point is that the size of the one-loop correction is controlled by the gauge beta function of heavy fields. This fact coincides with the spectrum of [14] which is determined by wavefunction renormalizations from vector messengers [15]. The dependence on the beta-function coefficients also appears in the supergravity models.

3.1.2 Matter contribution

Let us consider another one-loop contribution to the gaugino masses from matter multiplets. With the mass splitting between matter fermions and sfermions, the one-loop correction to the gaugino two-point function is evaluated as

$$I_{\text{mat}} = \left(\frac{\sqrt{2}g}{\sqrt{N}}\right)^2 T_2(R) \times\sum_{n=0}^{N-1}\int\frac{\mathrm{d}^4k}{\mathrm{i}(2\pi)^4} \quad (3.10)$$
$$\times\left[\frac{1}{m_n-\not{k}}\left(\frac{1/2}{m_{\phi_{n+}}^2-(p-k)^2}+\frac{1/2}{m_{\phi_{n-}}^2-(p-k)^2}\right)\right.$$

$$-\frac{1}{m_n+\not{k}}\left(\frac{1/2}{m_{\phi_{n+}}^2-(p-k)^2}+\frac{1/2}{m_{\phi_{n-}}^2-(p-k)^2}\right)\Bigg],$$

where $T_2(R)$ is the quadratic Dynkin index for the representation R of the unbroken gauge group G. The scalar mass eigenvalues $m_{\phi_{n\pm}}$ are given in (2.28) with $\Gamma=1/4$ (see the discussion in Sects. 2.2.2 and 2.2.3). The divergent part of (3.10) is given by

$$I_{\rm mat}^{\rm div}=\frac{2NT_2(R)}{16\pi^2}\left(\frac{g}{\sqrt{N}}\right)^2\not{p}\,\ln\Lambda^2. \tag{3.11}$$

Note that this is a supersymmetric correction. There is no divergence to the gaugino masses from the matter multiplets since only scalar components receive SUSY-breaking masses. In other words, the loop integrals converge if the scalar propagators are expanded with respect to SUSY-breaking VEVs. As for the finite part, we obtain

$$I_{\rm mat}^{\rm fin}=\frac{NT_2(R)}{16\pi^2}\left(\frac{g}{\sqrt{N}}\right)^2\frac{F_T}{\langle T\rangle}. \tag{3.12}$$

One can see that as in the vector case, the matter contribution is controlled by the modulus T. After all, the total amount of mass corrections is read from (3.9) and (3.12), and the gaugino mass up to one-loop level is given by

$$m_{\lambda_{0\pm}}=\pm\frac{F_{S5}}{2\langle S_5\rangle}-\frac{F_T}{2\langle T\rangle}+N\Big[-2C_2(G)+2T_2(R)\Big]\frac{g_4^2}{16\pi^2}\frac{F_T}{\langle T\rangle}. \tag{3.13}$$

The radiative correction is proportional to F_T and the gauge beta function of the heavy vector and matter multiplets.

3.2 Higgs masses

Next we study one-loop corrections to the masses of the Higgs scalars. Having the large top Yukawa coupling in mind, the corrections from various types of large Yukawa couplings will be investigate in detail. Gauge corrections can be estimated in a similar way and give qualitatively similar results. In the numerical evaluations, we will focus on the three special limits discussed in Sect. 2 and find that all types of radiative corrections to Higgs masses are consistent with the expected SUSY-breaking patterns. In particular, the corrections become ultraviolet finite in the limit of the radius modulus SUSY breaking. (We have already found in the previous section the same result for gaugino masses [see (3.7)].) The compactification-scale dependence of the radiative corrections is another important factor to be examined. We will show that its quantitative behavior is also properly taken into account in our framework.

3.2.1 Bulk–brane–brane couplings

We want to study the radiative corrections from Yukawa couplings involving the KK modes of the matter fields. The first case we consider is a Yukawa coupling among one "bulk" field and two "brane" fields. Suppose that L_i $(i=1,\dots,N)$ and $\bar{L}_j$ $(j=1,\dots,N-1)$ are introduced as matter multiplets as described in Sect. 2.2 and e as a chiral superfield charged only under G_1 (would-be localized field on a brane in the five-dimensional point of view). The transformation properties are listed below:

	G_1	G_2	G_3	$\dots$	G_N
L_1	$\square$	1	$\dots$	$\dots$	1
$\bar{L}_1$	$\bar{\square}$	1	$\dots$	$\dots$	1
L_2	1	$\square$	$\dots$	$\dots$	1
$\bar{L}_2$	1	$\bar{\square}$	$\dots$	$\dots$	1
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
e	$\bar{\square}$	1	$\dots$	$\dots$	1
H	1	1	$\dots$	$\dots$	1

(3.14)

In addition to the tree-level matter Lagrangian (2.17), we have a gauge-invariant Yukawa term among L_1, e and the Higgs field H. Other gauge-invariant couplings may be forbidden by symmetry arguments, for example, invariance under $e\to -e$ and $H\to -H$. The resultant superpotential is written in terms of the mass eigenstates of the L defined in Sect. 2

$$W=y\left(\sum_{n=0}^{N-1}\eta_n^L\widetilde{L}_n\right)eH+\sum_{n=0}^{N-1}m_{L_n}\widetilde{L}_n\widetilde{\bar{L}}_n, \tag{3.15}$$

where m_{L_n} are the KK masses (2.22), and η_n^L denotes the mixing between L_1 and the mass eigenstate $\widetilde{L}_n$. Its explicit form has been derived in Sect. 2.3:

$$\eta_n^L=\sqrt{\frac{2}{2^{\delta_{n0}}N}}\cos\left(\frac{n\pi}{2N}\right). \tag{3.16}$$

Eliminating the matter auxiliary fields from the Lagrangian (2.17) and (3.15) leads to

$$\begin{aligned}\mathcal{L}=&-y\psi_e\left(\sum_{n=0}^{N-1}\eta_n^L\psi_{\widetilde{L}_n}\right)H\\&-\frac{|y|^2}{2}\left(\left|\sum_{n=0}^{N-1}\eta_n^L\widetilde{L}'_n\right|^2+\left|\sum_{n=0}^{N-1}\eta_n^{L*}\widetilde{\bar{L}}'_n\right|^2\right)H^\dagger H\\&-\frac{y}{\sqrt{2}}eH\sum_{n=0}^{N-1}\eta_n^L\\&\times\left[\left(\frac{F_S}{2\langle S\rangle}-m_{L_n}^*\right)\widetilde{L}'_n+\left(\frac{F_S}{2\langle S\rangle}+m_{L_n}^*\right)\widetilde{\bar{L}}'^\dagger_n\right]\\&+\text{h.c.}+\dots,\end{aligned} \tag{3.17}$$

where ψ_X means the spinor component of a chiral multiplet X. The sfermion fields with primes are the mass eigenstates which completely diagonalize the mass matrix involving SUSY-breaking effects,

$$\widetilde{L}_n=\frac{1}{\sqrt{2}}\widetilde{L}'_n+\frac{1}{\sqrt{2}}\widetilde{\bar{L}}'^\dagger_n,\qquad \widetilde{\bar{L}}_n=\frac{-1}{\sqrt{2}}\widetilde{L}'_n+\frac{1}{\sqrt{2}}\widetilde{\bar{L}}'^\dagger_n. \tag{3.18}$$

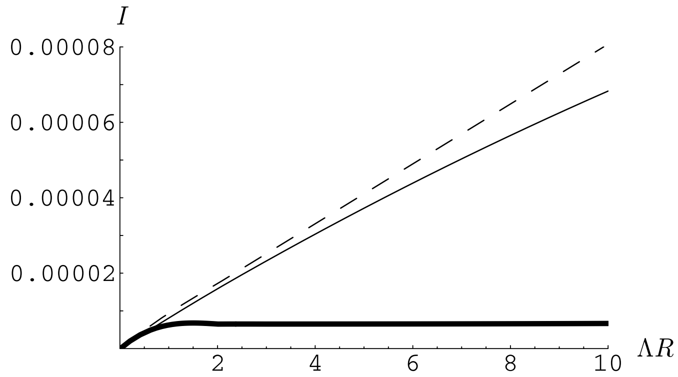

Fig. 1. The ultraviolet cutoff dependences of the loop correction from the bulk–brane–brane type Yukawa coupling. From the upper one, the dilaton, moduli and the radius modulus SUSY breaking are plotted. We take $y = 1$ and $N = 200$ as an example

It is important to notice that in deriving the Lagrangian (3.17), we have partly fixed the normalization function K_h for the matter multiplets as $\left\langle \frac{\partial \ln K_h}{\partial S} \right\rangle = \frac{1}{2\langle S \rangle}$. As will be seen below, this form of VEV is required for the model to be consistent at the quantum level. Together with (2.29), we obtain

$$K_h(S, S^\dagger) = c(S + S^\dagger), \tag{3.19}$$

with an arbitrary constant c.

With these matter–Higgs interactions at hand, we have three types of diagrams which contribute to the Higgs mass corrections. Each diagram involves an interaction in the Lagrangian (3.17):
(1) Yukawa couplings with the fermions running in the loop,
(2) sfermion-Higgs quartic couplings, and
(3) sfermion-Higgs trilinear couplings. These contributions to the Higgs two-point function become

$$I^{(1)} = -2|y|^2 \sum_{n=0}^{N-1} \int \frac{\mathrm{d}^4 p}{\mathrm{i}(2\pi)^4} \frac{|\eta_n^L|^2}{p^2 - m_{L_n}^2}, \tag{3.20}$$

$$I^{(2)} = |y|^2 \sum_{n=0}^{N-1} \int \frac{\mathrm{d}^4 p}{\mathrm{i}(2\pi)^4} \frac{|\eta_n^L|^2}{2} \times \left[\frac{1}{p^2 - m_{L_{n+}}^2} + \frac{1}{p^2 - m_{L_{n-}}^2} \right] + |y|^2 \int \frac{\mathrm{d}^4 p}{\mathrm{i}(2\pi)^4} \frac{1}{p^2}, \tag{3.21}$$

$$I^{(3)} = |y|^2 \sum_{n=0}^{N-1} \int \frac{\mathrm{d}^4 p}{\mathrm{i}(2\pi)^4} \frac{|\eta_n^L|^2}{2} \times \left[\frac{\left| \frac{F_S}{2\langle S \rangle} - m_{L_n}^* \right|^2}{p^2 - m_{L_{n+}}^2} + \frac{\left| \frac{F_S}{2\langle S \rangle} + m_{L_n}^* \right|^2}{p^2 - m_{L_{n-}}^2} \right] \frac{1}{p^2}. \tag{3.22}$$

The sfermion mass eigenvalues $m_{L_{n\pm}}^2$ are given by (2.28). We show in Fig. 1 the total one-loop correction $I = I^{(1)} + I^{(2)} + I^{(3)}$. The horizontal axis denotes the momentum cutoff Λ normalized by the compactification scale. We particularly focus on the three limits of the SUSY-breaking order parameters which were presented in Table 1. It is found from the figure that the loop correction is independent of the momentum cutoff and becomes finite in the limit of the radius modulus breaking (bold line). On the other hand, in the dilaton/moduli dominance scenarios (dashed/solid lines), the corrections linearly depend on Λ. This behavior is understood as the number of KK modes running around the loop. That is, the KK modes whose masses are below the momentum cutoff can contribute to the radiative corrections. In these three limited cases, the structure of the radiative corrections is consistent with the results which have been obtained in the literature.

The finite result for the Higgs mass is also analytically understood by noting that, in the limit $F_{S5}/\langle S_5 \rangle = F_S/\langle S \rangle + F_T/\langle T \rangle = 0$, the total correction I can simply be summarized as follows:

$$I\left(F_{S_5} = 0\right) = -|y|^2 \sum_{n=0}^{N-1} \sum_{i=+,-} \int \frac{\mathrm{d}^4 p}{\mathrm{i}(2\pi)^4} |\eta_n^L|^2 \left[\frac{1}{p^2 - m_{L_n}^2} - \frac{1}{p^2 - m_{L_{n\,i}}^2} \right]. \tag{3.23}$$

The ultraviolet finiteness of this type of momentum integral will be discussed in Sect. 5.

3.2.2 Bulk–bulk–brane couplings

In a similar way, we calculate one-loop Higgs masses from the bulk–bulk–brane-type Yukawa couplings. In this case, the matter multiplets L_i ($\bar{L}_i$) and e_j ($\bar{e}_j$) are charged under the G gauge symmetry, and the Higgs H is introduced as a singlet field. To implement the orbifold projection, L_N and $\bar{e}_1$ are removed from the spectrum. The transformation properties of these fields are given by

	G_1	G_2	G_3	…	G_N
L_1	$\square$	1	…	…	1
$\bar{L}_1$	$\bar{\square}$	1	…	…	1
L_2	1	$\square$	…	…	1
$\bar{L}_2$	1	$\bar{\square}$	…	…	1
⋮	⋮	⋮	⋮	⋮	⋮
e_1	$\bar{\square}$	1	…	…	1
e_2	1	$\bar{\square}$	…	…	1
$\bar{e}_2$	1	$\square$	…	…	1
⋮	⋮	⋮	⋮	⋮	⋮
H	1	1	…	…	1

(3.24)

The superpotential terms for masses and Yukawa couplings are written in the mass eigenstate basis defined in a similar fashion to (3.18),

$$W = y \left(\sum_{n=0}^{N-1} \eta_n^L \widetilde{L}_n \right) \left(\sum_{l=0}^{N-1} \eta_l^e \widetilde{e}_l \right) H + \sum_{n=0}^{N-1} m_{L_n} \widetilde{L}_n \widetilde{\bar{L}}_n$$

$$+\sum_{n=0}^{N-1} m_{e_n}\widetilde{e}_n\widetilde{\overline{e}}_n. \tag{3.25}$$

Notice that we have introduced only one type of Yukawa term yL_1e_1H. This is analogous to the fact that a bulk Yukawa coupling is not consistent with the symmetries of five-dimensional supersymmetric theories. In the present four-dimensional model, other gauge-invariant Yukawa terms can be forbidden by some discrete symmetry, as already done for the bulk–brane–brane type Yukawa terms. Integrating out the matter multiplet F terms, we obtain the following interaction terms

$$\begin{aligned}\mathcal{L} = &-\frac{|y|^2}{2}\\ &\times\left[\left|\sum_{n=0}^{N-1}\eta_n^L\widetilde{L}'_n\right|^2+\left|\sum_{n=0}^{N-1}\eta_n^{L*}\widetilde{\overline{L}}'_n\right|^2+\left|\sum_{n=0}^{N-1}\eta_n^e\widetilde{e}'_n\right|^2\right.\\ &\left.+\left|\sum_{n=0}^{N-1}\eta_n^{e*}\widetilde{\overline{e}}'_n\right|^2\right]H^\dagger H\\ &-\frac{y}{2}H\sum_{n=0}^{N-1}\sum_{l=0}^{N-1}\eta_n^e\eta_l^L(\widetilde{e}'_n-\widetilde{\overline{e}}'^{\dagger}_n)\\ &\times\left[\left(\frac{F_S}{2\langle S\rangle}-m^*_{L_l}\right)\widetilde{L}'_l+\left(\frac{F_S}{2\langle S\rangle}+m^*_{L_l}\right)\widetilde{\overline{L}}'^{\dagger}_l\right]\\ &-\frac{y}{2}H\sum_{n=0}^{N-1}\sum_{l=0}^{N-1}\eta_n^L\eta_l^e(\widetilde{L}'_n-\widetilde{\overline{L}}'^{\dagger}_n)\\ &\times\left[\left(\frac{F_S}{2\langle S\rangle}-m^*_{e_l}\right)\widetilde{e}'_l+\left(\frac{F_S}{2\langle S\rangle}+m^*_{e_l}\right)\widetilde{\overline{e}}'^{\dagger}_l\right] \quad (3.26)\\ &-y\left(\sum_{n=0}^{N-1}\eta_n^e\psi_{\widetilde{e}_n}\right)\left(\sum_{l=0}^{N-1}\eta_l^L\psi_{\widetilde{L}_l}\right)H+\text{h.c.}+\ldots\end{aligned}$$

With this Lagrangian, the diagrams of loop corrections to the Higgs mass are similar to those in the previous section:
(1) Yukawa couplings to the Higgs,
(2) sfermion–Higgs quartic couplings, and
(3) sfermion–Higgs trilinear couplings. Each contribution is given by

$$\begin{aligned}I^{(1)} &= -2|y|^2\sum_{n=0}^{N-1}\sum_{l=0}^{N-1}\int\frac{\mathrm{d}^4p}{\mathrm{i}(2\pi)^4}|\eta_n^L|^2|\eta_l^e|^2\\ &\quad\times\frac{p^2}{(\not p-m_{L_n})(\not p-m_{e_l})},\end{aligned} \tag{3.27}$$

$$\begin{aligned}I^{(2)} &= |y|^2\sum_{n=0}^{N-1}\int\frac{\mathrm{d}^4p}{\mathrm{i}(2\pi)^4}\frac{|\eta_n^L|^2}{2}\\ &\quad\times\left[\frac{1}{p^2-m^2_{L_{n+}}}+\frac{1}{p^2-m^2_{L_{n-}}}\right]+(L\leftrightarrow e),\end{aligned} \tag{3.28}$$

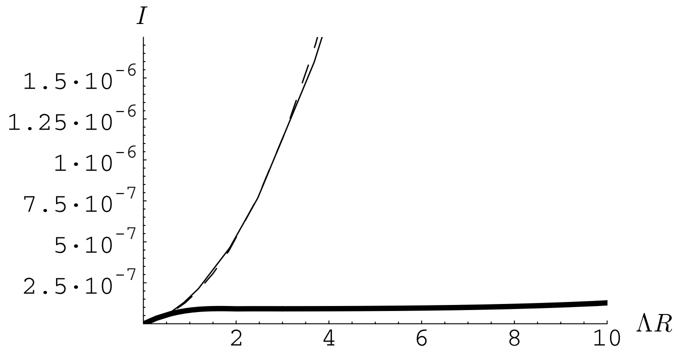

Fig. 2. The cutoff dependences of the loop correction from the bulk–bulk–brane-type Yukawa coupling. The dilaton, moduli and radius modulus SUSY breaking are plotted from the above (the dashed, solid, and bold lines, respectively). We take $y=1$ and $N=200$ as an example

$$\begin{aligned}I^{(3)} &= |y|^2\sum_{n=0}^{N-1}\sum_{l=0}^{N-1}\int\frac{\mathrm{d}^4p}{\mathrm{i}(2\pi)^4}\frac{|\eta_n^L|^2}{2}\frac{|\eta_l^e|^2}{2}\\ &\quad\times\left[\frac{1}{p^2-m^2_{e_{n+}}}+\frac{1}{p^2-m^2_{e_{n-}}}\right]\\ &\quad\times\left[\frac{\left|\frac{F_S}{2\langle S\rangle}-m^*_{L_n}\right|^2}{p^2-m^2_{L_{n+}}}+\frac{\left|\frac{F_S}{2\langle S\rangle}+m^*_{L_n}\right|^2}{p^2-m^2_{L_{n-}}}\right]+(L\leftrightarrow e).\end{aligned} \tag{3.29}$$

The total loop correction $I=I^{(1)}+I^{(2)}+I^{(3)}$ is plotted for several cases in Fig. 2, where the Yukawa coupling constant y is taken to be 1 as an example. One can see that the cutoff dependences of the loop correction have similar behaviors as in the previous section; I is insensitive to the momentum cutoff Λ for the radius modulus SUSY breaking, while I is cutoff dependent (roughly proportional to Λ^2) for the other cases, which dependence is interpreted as the number of KK modes circulating in the loops.

3.2.3 Bulk–bulk–bulk couplings

Finally we consider the case that all L, e and H are introduced as bulk multiplets and examine radiative corrections to the massless Higgs scalar. Suppose that corrections come from the superpotential Yukawa coupling $L_1e_1H_1$ and other Yukawa terms are absent. Thus the situation is almost the same as the bulk–bulk–brane Yukawa case in Sect. 3.2.2, except for an overall rescaling of the Yukawa couplings. However we now have an additional scalar three-point vertex, which is a cross term generated by integrating out the F component of the Higgs multiplet,

$$\begin{aligned}&-y\sum_{n=0}^{N-1}\eta_n^H\left(\left\langle\frac{\partial\ln K_h}{\partial\ln S}\right\rangle\frac{F_S}{\langle S\rangle}\widetilde{H}_n+m^*_{H_n}\widetilde{\overline{H}}^{\dagger}_n\right)\\ &\quad\times\left(\sum_{n=0}^{N-1}\eta_n^L\widetilde{L}_n\right)\left(\sum_{l=0}^{N-1}\eta_l^e\widetilde{e}_l\right)+\text{h.c.}\end{aligned} \tag{3.30}$$

Other couplings of the Higgs zero mode are modified only by the rescaling with the wavefunctions $\eta_0^{H,\bar{H}} = 1/\sqrt{N}$, which does not cause any qualitative changes to the mass corrections.

While the corrections induced by the vertex (3.30) are generally divergent logarithmically, no qualitative change compared to the bulk–bulk–brane Yukawa case is found for the supergravity SUSY-breaking models. On the other hand, one might wonder that the vertex (3.30) gives rise to a cutoff-dependent contribution for the radius modulus breaking where even logarithmic divergences are found to vanish in the previous analyses. However, it is found from the matter Kähler form (3.19) that (3.30) leads to the interactions of the Higgs $n = 0$ modes

$$-\frac{y}{\sqrt{2N}}\left(m_{H_{0+}}\widetilde{H}_0' + m_{H_{0-}}\widetilde{\bar{H}}_0'^{\dagger}\right)\left(\sum_{n=0}^{N-1}\eta_n^L\widetilde{L}_n\right)\left(\sum_{l=0}^{N-1}\eta_l^e\widetilde{e}_l\right) + \text{h.c.}, \qquad (3.31)$$

in the $F_{S5} = 0$ limit. Therefore the coupling of the Higgs massless mode of (3.30) is vanishing, independently of which of the Higgs fields H and $\bar{H}$ contain a light mode. As a result, the radiative corrections from bulk–bulk–bulk Yukawa couplings are qualitatively unchanged from the bulk–bulk–brane Yukawa case.

3.3 Radius dependence

We have so far analyzed the momentum cutoff dependence of the radiative corrections. As another important factor, let us examine how the loop corrections to the scalar masses depend on the compactification radius R of the fifth dimension. Consider for example the corrections from a bulk–brane–brane-type Yukawa coupling. When all the parameters except for R are fixed, the following R dependences of the one-loop corrections on the scalar masses are expected:

$$\delta m_\phi^2 \propto R \qquad \text{[dilaton]}, \qquad (3.32)$$

$$\delta m_\phi^2 \propto R^3 \qquad \text{[moduli]}, \qquad (3.33)$$

$$\delta m_\phi^2 \propto R^{-2} \qquad \text{[radion]}. \qquad (3.34)$$

These behaviors can be understood as follows. In the dilaton dominant case, the R^1 behavior is interpreted as the number of KK modes ($\simeq \Lambda R$) propagating in the loop diagrams. In the moduli dominant case, we have an R^3 factor from the number of KK modes (R^1) and SUSY-breaking effects $[(F_T/\langle T\rangle)^2 \propto R^2]$. On the other hand, the radiative corrections to the Higgs masses are expected to behave like R^{-2} in the radius modulus breaking. This is because the radius modulus breaking is equivalent to the boundary condition breaking of supersymmetry [17], and hence quadratic divergences are cut off by the compactification scale due to the locality in the extra dimension.

In Fig. 3, we show the radius dependences of the scalar mass correction from bulk–brane–brane Yukawa couplings (Sect. 3.2.1). The horizontal axis denotes the size of R and the vertical one the magnitudes of the one-loop corrections. In the figure, R is taken to be within a reasonable regime, $1 < \Lambda R < 10$ (Λ is the momentum cutoff). The upper bound comes from the fact that finite scalar masses are realized in the region $\Lambda \ll v$, and the lower bound from the requirement that at least one KK mode runs around the loop. One can see from the figure that the expected results (3.32)–(3.34) are certainly reproduced in our model.

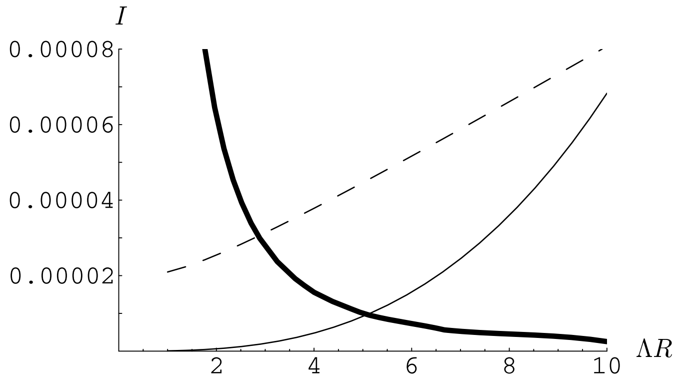

Fig. 3. Typical radius dependences of the radiative correction to the Higgs mass. The horizontal axis means the compactification radius and the vertical one the magnitudes of the correction. The dashed, solid, and bold lines correspond to the dilaton dominance, the moduli dominance, and the radius modulus breaking, respectively. We take in the figure $y = 1$ and $N = 200$

Summarizing Sect. 3, we have explicitly calculated one-loop radiative corrections to the gaugino and Higgs scalar masses. The one-loop gaugino masses are proportional to the gauge beta function of the heavy fields and also to the modulus auxiliary VEV F_T. This seems to be a result consistent with the predictions of string-inspired supergravity models. For the Higgs mass, we have evaluated the one-loop contributions from various types of Yukawa interactions. We have particularly found that in the limit of the radius modulus SUSY breaking, the corrections are insensitive to the ultraviolet cutoff of momentum integrals, while those related to the supergravity models depend linearly or quadratically on the cutoff scale. Furthermore, the radius R dependence of the Higgs mass has been studied. There, we have found that even for a finite number of gauge groups – that is, with finite KK modes included – the expected behaviors emerge in certain regions of the modulus F terms.

4 Finiteness of radiative corrections

4.1 Large N

We have observed by examining the cutoff and compactification-scale dependences that finite radiative corrections appear in the limit of the radius modulus breaking for a finite number of gauge groups (i.e. finite KK modes). In continuous five-dimensional theory, radiative corrections to the Higgs masses are found to become finite if one adopts SUSY breaking with boundary conditions [18]. In this case, it is important to include an infinitely large number of KK modes in summing up loop corrections while preserving five-dimensional supersymmetry. In other words, if one is allowed to include the effects of KK modes sufficiently heavier than the momentum cutoff, the locality in the extra dimension is recovered [19]. In our model, this is interpreted as involving a large number of gauge groups or taking a lower four-dimensional cutoff. For example, for $N = 200$, loop corrections become cutoff independent in the regime $\Lambda R < 15$, while the cutoff insensitivity can hardly be seen for $N = 10$. However we here want to stress that to have finite corrections is not the main point of this work because we only deal with a four-dimensional theory. (For an approach to cutoff-insensitive Higgs masses in product-group gauge theories, see [20].)

In this subsection, we analytically clarify the high-energy behavior of the radiative corrections to Higgs scalar masses, especially focusing on the insensitivity to an ultraviolet cutoff. Let us study the following typical form of radiative corrections to the Higgs masses from bosonic and fermionic KK modes:

$$I_b = |y|^2 \sum_{n,\pm} \int \frac{\mathrm{d}^4 p}{(2\pi)^4} \frac{1}{p^2 + m_{n\pm}^2},$$
$$I_f = -2|y|^2 \sum_{n} \int \frac{\mathrm{d}^4 p}{(2\pi)^4} \frac{1}{p^2 + m_{n}^2}, \tag{4.1}$$

where the factor 2 in I_f has been introduced so that the boson and fermion degrees of freedom are equal. This type of corrections would appear, for example, from the Yukawa couplings between bulk matter multiplets and a boundary Higgs field. In fact, we have already encountered it in (3.23). (That is the reason why we have written the coupling as y in the above equations.) A similar analysis can also be performed for the gauge interactions. We treat these momentum integrals with the proper-time regularization and then obtain for I_b

$$I_b = \frac{|y|^2}{16\pi^2} \sum_{n,\pm} \int_0^\infty \mathrm{d}t \, \frac{1}{t^2} \mathrm{e}^{-t m_{n\pm}^2}, \tag{4.2}$$

and a similar expression for I_f. An important point is that the integration and summation can be safely exchanged if one introduces a ultraviolet momentum cutoff Λ and truncates the KK-mode sum at a finite level. Note that the model in this paper naturally contains a finite number of KK modes. Thus we find

$$I_b = \frac{|y|^2}{16\pi^2} \int_{1/\Lambda^2}^\infty \mathrm{d}t \, \frac{1}{t^2} \sum_{n,\pm} \mathrm{e}^{-t m_{n\pm}^2}. \tag{4.3}$$

To see the cutoff dependence of the radiative corrections, we examine the beta functions in the sense of the Wilson renormalization-group equations:

$$\frac{\partial I_{b\,(f)}}{\partial \ln \Lambda^2} = \frac{\epsilon_{b\,(f)}\,|y|^2}{16\pi^2} \Lambda^2, \tag{4.4}$$

$$\epsilon_b = \sum_{n,\pm} \mathrm{e}^{-m_{n\pm}^2/\Lambda^2}, \quad \epsilon_f = 2\sum_{n} \mathrm{e}^{-m_n^2/\Lambda^2}. \tag{4.5}$$

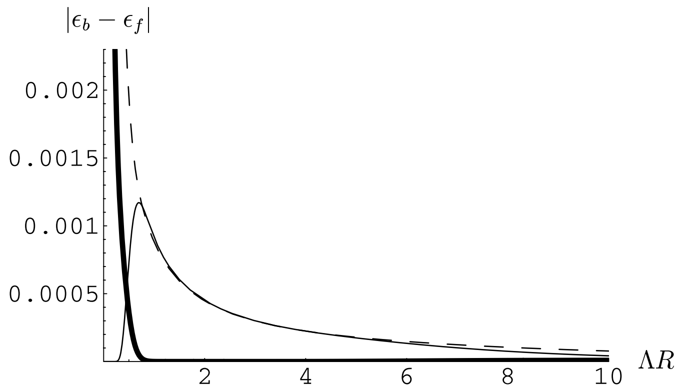

Fig. 4. The cutoff dependences of radiative corrections in the large N limit. The horizontal axis denotes the cutoff scale and the vertical one the difference between ϵ_b and ϵ_f. We take $F_S/\langle S\rangle$ and/or $F_T/\langle T\rangle = 1/10R$ in the figure. The dashed, solid, and bold lines correspond to the dilaton, moduli dominant, and the radius modulus SUSY breaking, respectively

Since we are now interested in a particular relation between finite radiative corrections and the radius modulus breaking, it is appropriate to take the large N limit (the five-dimensional limit) in analyzing the corrections. In this limit, the bosonic and fermionic KK modes are found from the previous results to have the following form of the mass eigenvalues:

$$m_{n\pm}^2 = \left(\frac{n}{R} \pm \frac{F_T}{2\langle T\rangle}\right)^2 + \left|\frac{F_S}{2\langle S\rangle}\right|^2 - \left|\frac{F_T}{2\langle T\rangle}\right|^2,$$
$$m_n^2 = \left(\frac{n}{R}\right)^2, \tag{4.6}$$

which is valid for vector and matter multiplets up to the second order of the F (or exact in the dilaton/radius modulus SUSY-breaking cases). With this mass splitting, we show in Fig. 4 the differences between the beta-function coefficients ϵ_b and ϵ_f, i.e. the cutoff dependences of the radiative corrections for the three limited cases discussed before. It is clearly seen from the figure that the cutoff-dependent behaviors are rather different from each other; in particular, the ultraviolet divergence is highly suppressed in the radius modulus breaking case. The dilaton and moduli SUSY breaking have a similar behavior for a large value of the cutoff scale Λ. This is because in both cases, the corrections are proportional to the number of KK modes as mentioned previously.

To analytically study these divergence properties, we estimate the contributions by use of the Poisson formula and obtain

$$\epsilon_b = 2\sqrt{\pi}\Lambda R \times \exp\left[\frac{-1}{4\Lambda^2}\left(\left|\frac{F_S}{\langle S\rangle}\right|^2 - \left|\frac{F_T}{\langle T\rangle}\right|^2\right)\right] \times \sum_n \mathrm{e}^{-\frac{\pi^2 n^2}{(\Lambda R)^2}} \cos(\pi F_T R^2 n), \tag{4.7}$$

$$\epsilon_f = 2\sqrt{\pi}\Lambda R \sum_n \mathrm{e}^{-\frac{n^2}{(\Lambda R)^2}} . \tag{4.8}$$

As can be seen in this expression, for a large value of ΛR, the zero-mode contributions dominate due to the exponential damping factors. If assumed that the SUSY-breaking scale is much lower than the compactification radius, the zero-mode contribution approximately gives

$$\epsilon = \epsilon_b^0 - \epsilon_f^0 \simeq \frac{\sqrt{\pi}}{2}\frac{R}{\Lambda}\left(\left|\frac{F_T}{\langle T\rangle}\right|^2 - \left|\frac{F_S}{\langle S\rangle}\right|^2 \right) . \tag{4.9}$$

It is easily found that the Λ^{-1} dependence of (4.9) implies linearly divergent corrections to Higgs masses. We can see from (4.9) that in the radius modulus breaking case, where $F_{S5} \propto (F_S/\langle S\rangle + F_T/\langle T\rangle) = 0$, the radiative correction becomes cutoff insensitive and therefore a finite result follows. On the other hand, the dilaton/moduli dominant cases generally have divergent corrections proportional to ΛR.

4.2 Boundary condition breaking

In continuum five-dimensional theories, it is well known that supersymmetry breaking with boundary conditions gives finite radiative corrections to scalar soft masses [18]. This is partly because supersymmetry is broken globally in the bulk and hence the breaking effect is cut off by the size of the extra dimension. On the other hand, in the present four-dimensional model, the finiteness is achieved on a specific line of the SUSY-breaking parameter space. We will show how these two cases are related. To this end, we formulate the "boundary" condition breaking of supersymmetry in our setup, and examine whether the resulting spectrum matches that of the radius modulus breaking ($F_{S5} = 0$). If these two spectra are found to be equivalent, they may be transformed to each other by a unitary transformation, which gives a support that radiative corrections are ultraviolet finite.

4.2.1 Without twists

Before discussing the boundary condition breaking, it is useful to recall how to compactify the latticized extra dimensions. To begin with, suppose that there are an infinite number of gauge groups and corresponding matter fields. Compactifying physical space is performed with the identification of indices under $i \to i + N$, which is interpreted as a coordinate translation in the extra dimension. On the other hand, the identification on the field space, i.e. boundary conditions under this translation, is given by $\Phi_{i+N} \equiv \Phi_i$. These procedures result in leaving the N copies of gauge groups and matter fields as physical degrees of freedom. The identification on the field space is clearly seen by recombining the fields as follows:

$$\Phi_j = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} (\omega_k)^j \, \widetilde{\Phi}_k , \tag{4.10}$$

where $\omega_k \equiv \mathrm{e}^{\frac{2\pi k}{N}\mathrm{i}}$, which is just the Fourier mode expansion in the continuum limit. Note that there are only finite numbers of mass eigenstates $\widetilde{\Phi}_k$, because the model comes back to higher dimensions only in the large N limit. At this stage, it is straightforward to include various types of twisted boundary conditions. In what follows, we will study a specific case which is related to the radius modulus breaking of supersymmetry.

4.2.2 $SU(2)$ twist

Along the line discussed above, we demonstrate the twisted boundary condition for vector multiplets as an illustrative example. The field content we consider is given by Table 1 except that we now have an infinite numbers of gauge groups and Q fields. In particular, there are two types of spinors λ and χ for every gauge group. The identification of physical space is the same as the above toy model. A new ingredient now introduced is the boundary condition on the field space. Let us consider the following $SU(2)$ twist upon the identification of field space:

$$\begin{pmatrix} \lambda_{j+N} \\ \chi_{j+N} \end{pmatrix} = \mathrm{e}^{2\pi\mathrm{i}\,\theta_a\sigma_a} \begin{pmatrix} \lambda_j \\ \chi_j \end{pmatrix} , \tag{4.11}$$

where σ_a are the Pauli matrices. The parameters θ_a will turn out to be SUSY-breaking order parameters. The mass matrix of these spinor fields must be consistent with this identification and therefore takes the form for $\theta_a \ll 1$

$$v\big(\lambda \mid \chi\big)(M_0 + \delta M) \begin{pmatrix} \lambda \\ \chi \end{pmatrix} + \text{h.c.}, \tag{4.12}$$

$$M_0 = \left(\begin{array}{cccc|cccc} & & & & 1 & & & -1 \\ & & & & -1 & \ddots & & \\ & & & & & \ddots & \ddots & \\ & & & & & & -1 & 1 \\ \hline 1 & -1 & & & & & & \\ & \ddots & \ddots & & & & & \\ & & \ddots & -1 & & & & \\ -1 & & & 1 & & & & \end{array}\right) , \tag{4.13}$$

$$\delta M = \left(\begin{array}{ccc|ccc} & & & & & -2\pi\mathrm{i}\theta_3 \\ & & & & & \\ \hline & & & & & -2\pi(\mathrm{i}\theta_1 + \theta_2) \\ & & & & & \\ -2\pi\mathrm{i}\theta_3 & & & -2\pi(\mathrm{i}\theta_1 + \theta_2) & & \end{array}\right) . \tag{4.14}$$

The M_0 part denotes the mass matrix for untwisted fields, whose eigenvalues are given by (2.4). On the other hand, δM arises due to the twisting boundary condition (4.11).

As we mentioned before, these mass terms become the kinetic energy along the extra dimension if one takes the continuum limit.

Let us diagonalize the mass matrix by perturbation theory for $\theta_a \ll 1$. The eigenvalues and eigenmodes of the unperturbed matrix M_0 are given by

$$m_{k\pm}^{(0)} = \pm 2v \sin\left(\frac{k\pi}{N}\right),$$

$$\widetilde{\psi}_k^{\pm} = \frac{1}{\sqrt{2N}} \sum_{j=1}^{N} (\omega_k)^j (\pm\lambda_j + \chi_j). \tag{4.15}$$

Irrelevant overall phases factors have been absorbed into the field redefinition. In perturbation theory, the non-zero matrix elements of δM linear in the θ are

$$\left\langle \widetilde{\psi}_k^+ \right| \delta M \left| \widetilde{\psi}_k^+ \right\rangle = \left\langle \widetilde{\psi}_k^- \right| \delta M \left| \widetilde{\psi}_k^- \right\rangle \tag{4.16}$$
$$= \frac{-2\pi}{N} \cos\left(\frac{2\pi k}{N}\right) (\mathrm{i}\theta_1 + \theta_2 + \mathrm{i}\theta_3),$$

and the eigenvalues at first order are thus given by

$$m_{k\pm}^{(1)} = m_{k\pm}^{(0)} - \frac{2\pi v}{N} \cos\left(\frac{2\pi k}{N}\right) (\mathrm{i}\theta_1 + \theta_2 \pm \mathrm{i}\theta_3). \tag{4.17}$$

The first term on the right-handed side means the KK masses and therefore the second one is interpreted as a SUSY-breaking effect. For example, in the case that $\theta_1, \theta_2 \neq 0$ and $\theta_3 = 0$, the mass eigenvalues for the low-lying modes ($k \ll N$) read

$$m_{k\pm}^{(1)} \simeq \pm\frac{k}{R} - \frac{F_T}{2R}, \tag{4.18}$$

where the SUSY-breaking order parameter is identified as $F_T \equiv 2(\mathrm{i}\theta_1 + \theta_2)$. This is indeed the spectrum that is given by the radius modulus breaking (2.15). Therefore the finiteness property of radiative corrections in this case is now easily understood.

For $\theta_3 = 0$, half of the supersymmetry in five dimensions is left unbroken, namely, a linear combination of the zero-mode gauge fermions is massless, which composes a four-dimensional massless vector multiplet together with the gauge boson zero mode. When θ_3 is turned on as well as $\theta_{1,2}$, the remaining $N = 1$ supersymmetry is broken. This corresponds to applying the twisted boundary condition with a $U(1)$ rotation which does not commute with the $N = 1$ supersymmetry.

It may be interesting to rewrite the mass term (4.12) in terms of the fields with untwisted boundary conditions. We consider for simplicity the twisting with σ_2. It is straightforward to generalize the following result to the cases with generic types of twisting. Let us transform the spinor fields by

$$\begin{pmatrix} \lambda_j \\ \chi_j \end{pmatrix} = \mathrm{e}^{2\pi\mathrm{i}\theta_2 \frac{j}{N}\sigma_2} \begin{pmatrix} \lambda'_j \\ \chi'_j \end{pmatrix}. \tag{4.19}$$

It can be easily checked that the fields with primes satisfy the untwisted boundary condition under the translation $j \to j + N$,

$$\begin{pmatrix} \lambda'_{j+N} \\ \chi'_{j+N} \end{pmatrix} = \begin{pmatrix} \lambda'_j \\ \chi'_j \end{pmatrix}. \tag{4.20}$$

Almost all terms in the Lagrangian are invariant under this field redefinition, while the mass term (4.12) is not. In other words, this means that the kinetic term along the fifth dimension is not invariant under the coordinate-dependent phase rotation. In the basis of λ' and χ', the mass matrix becomes for $\theta \ll 1$,

$$(4.12) \simeq \left(\lambda' \mid \chi'\right) M_0 \begin{pmatrix} \lambda' \\ \chi' \end{pmatrix} \tag{4.21}$$
$$-\frac{2\pi\theta_2 v}{N} \left(\lambda'_1 \ldots \lambda'_N\right) \begin{pmatrix} & 1 & & \\ & & \ddots & \\ & & & 1 \\ 1 & & & \end{pmatrix} \begin{pmatrix} \lambda'_1 \\ \vdots \\ \vdots \\ \lambda'_N \end{pmatrix}$$
$$-\frac{2\pi\theta_2 v}{N} \left(\chi'_1 \ldots \chi'_N\right) \begin{pmatrix} & 1 & & \\ & & \ddots & \\ & & & 1 \\ 1 & & & \end{pmatrix} \begin{pmatrix} \chi'_1 \\ \vdots \\ \vdots \\ \chi'_N \end{pmatrix}.$$

Diagonalizing this matrix should lead to the eigenvalues (4.18) with $\theta_1 = 0$. The first term in the right-handed side is the kinetic energy term along the fifth dimension. The second and third terms can be interpreted as the mass terms which come from a VEV of the extra component of the gauge field [21]. That is, these three terms together make up the covariant derivative of the spinor fields. It has been shown [17] that the radius modulus breaking of supersymmetry is equivalent to the Wilson line breaking, proving that a VEV of the extra component of the graviphoton field generates SUSY breaking in five-dimensional off-shell supergravity (as seen in the second and third terms above). The present analysis makes it clear from a four-dimensional viewpoint that the relevant VEV is that of the modulus Q ($F_Q \propto F_T$).

5 Summary

In this paper, we have formulated four-dimensional SUSY breaking in product-group gauge theories. The model contains several modulus fields corresponding to such ones as the dilaton and the radion. The modulus fields satisfy the specific relations suggested by some correspondences to higher-dimensional physics. From these, the relations are extracted for the F component VEVs of the modulus fields. The non-trivial moduli dependences of the action have also been determined. We have shown that at intermediate energy regime, the mass spectra of typical SUSY breaking scenarios (the dilaton/moduli dominance and the radius modulus breaking) appear in the corresponding limits on the space of SUSY-breaking order parameters. We have calculated in detail the gaugino and Higgs scalar masses up to one-loop level. Our results seem to be consistent with various aspects of bulk SUSY breaking, e.g. string-inspired supergravity models.

The cutoff dependences of the loop corrections have been investigated in detail. We have calculated the gaugino and Higgs mass corrections from various types of gauge and Yukawa couplings. In particular, we have shown that insensitivity to an ultraviolet cutoff emerges for the radius modulus breaking case. However, for other cases, the spectrum depends linearly or quadratically on the momentum cutoff. This can be understood as the number of KK modes running in the loop diagrams. The compactification radius dependence of the one-loop mass spectrum has also been studied.

The finiteness property of radiative corrections has been examined from several different viewpoints. In particular, we have formulated in our setup the boundary condition breaking of supersymmetry (or the Hosotani mechanism) and have shown that the spectrum indeed agrees with that of the radius modulus breaking case. This result indicates that the obtained finite corrections are due to a global breaking of symmetries in the bulk.

While, in this work, we have focused on the special limits corresponding to higher-dimensional SUSY breaking, it will be interesting to investigate other regions of the modulus F terms. Such a generic pattern of F terms might induce sparticle mass spectra not yet explored in the literature. It is also possible to extend the present analysis to include brane matter which is interpreted as being charged under one of the gauge groups G_i. The SUSY-breaking masses of this type of fields depend on whether they can couple to the moduli S and Q. However, there seem to be, in general, few principles to fix the modulus couplings of the brane fields, and the couplings would also depend on more fundamental theories. The presence of brane fields may be useful to generate a Yukawa hierarchy of quarks and leptons. Explicit four-dimensional model construction along this line leads to concrete predictions of the (super) particle spectrum, that can be compared to those of other models. The model parameters might then be constrained by clarifying the spectrum and applying it to supersymmetric standard models, etc. We leave such phenomenological analyses to future work.

Acknowledgements. The authors would like to thank Tatsuo Kobayashi for collaboration at an early stage of this work and lots of valuable discussions. This work is supported in part by the Special Postdoctoral Researchers Program at RIKEN.

References

1. I. Antoniadis, Phys. Lett. B **246**, 377 (1990); N. Arkani-Hamed, S. Dimopoulos, G. Dvali, Phys. Lett. B **429**, 263 (1998); I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos, G. Dvali, Phys. Lett. B **436**, 257 (1998)
2. L. Randall, R. Sundrum, Phys. Rev. Lett. **83**, 3370 (1990); **83**, 4690 (1999)
3. N. Arkani-Hamed, A.G. Cohen, H. Georgi, Phys. Rev. Lett. **86**, 4757 (2001)
4. C.T. Hill, S. Pokorski, J. Wang, Phys. Rev. D **64**, 105005 (2001)
5. H.C. Cheng, C.T. Hill, S. Pokorski, J. Wang, Phys. Rev. D **64**, 065007 (2001); A. Sugamoto, Prog. Theor. Phys. **107**, 793 (2002); N. Arkani-Hamed, A.G. Cohen, H. Georgi, Phys. Lett. B **513**, 232 (2001); H.C. Cheng, C.T. Hill, J. Wang, Phys. Rev. D **64**, 095003 (2001); M. Bander, Phys. Rev. D **64**, 105021 (2001); C. Csaki, G.D. Kribs, J. Terning, Phys. Rev. D **65**, 015004 (2002); H.C. Cheng, K.T. Matchev, J. Wang, Phys. Lett. B **521**, 308 (2001); N. Arkani-Hamed, A.G. Cohen, H. Georgi, hep-th/0108089; C.T. Hill, Phys. Rev. Lett. **88**, 041601 (2002); N. Arkani-Hamed, A.G. Cohen, H. Georgi, JHEP **0207**, 020 (2002); I. Rothstein, W. Skiba, Phys. Rev. D **65**, 065002 (2002); P.H. Chankowski, A. Falkowski, S. Pokorski, JHEP **0208**, 003 (2002); N. Arkani-Hamed, A.G. Cohen, D.B. Kaplan, A. Karch, L. Motl, JHEP **0301**, 083 (2003); W. Skiba, D. Smith, Phys. Rev. D **65**, 095002 (2002); R.S. Chivukula, H.J. He, Phys. Lett. B **532**, 121 (2002); N. Arkani-Hamed, A.G. Cohen, T. Gregoire, J.G. Wacker, JHEP **0208**, 020 (2002); Z. Berezhiani, A. Gorsky, I.I. Kogan, JETP Lett. **75**, 530 (2002); C.T. Hill, A.K. Leibovich, Phys. Rev. D **66**, 016006 (2002); N. Arkani-Hamed, A. Cohen, E. Katz, A.E. Nelson, T. Gregoire, J. Wacker, JHEP **0208**, 021 (2002); A. Falkowski, H.D. Kim, JHEP **0208**, 052 (2002); C. Csaki, J. Erlich, J. Terning, Phys. Rev. D **67**, 025019 (2003); L. Randall, Y. Shadmi, N. Weiner, JHEP **0301**, 055 (2003); S. Chang, H. Georgi, hep-th/0209038; N. Arkani-Hamed, H. Georgi, M.D. Schwartz, hep-th/0210184; A. Falkowski, H.P. Nilles, M. Olechowski, S. Pokorski, hep-th/0212206; J. Giedt, E. Poppitz, M. Rozali, JHEP **0303**, 035 (2003); N. Arkani-Hamed, M.D. Schwartz, hep-th/0302110; M.D. Schwartz, hep-th/0303114
6. T. Kobayashi, N. Maru, K. Yoshioka, Eur. Phys. J. C **29**, 277 (2003)
7. K. Sfetsos, Nucl. Phys. B **612**, 191 (2001)
8. H. Abe, T. Kobayashi, N. Maru, K. Yoshioka, Phys. Rev. D **67**, 045019 (2003)
9. C. Csaki, J. Erlich, V.V. Khoze, E. Poppitz, Y. Shadmi, Y. Shirman, Phys. Rev. D **65**, 085033 (2002)
10. C. Csaki, J. Erlich, C. Grojean, G. Kribs, Phys. Rev. D **65**, 015003 (2002); H.C. Cheng, D.E. Kaplan, M. Schmaltz, W. Skiba, Phys. Lett. B **515**, 395 (2001)
11. L.E. Ibáñez, D. Lüst, Nucl. Phys. B **382**, 305 (1992); B. de Carlos, J.A. Casas, C. Muñoz, Phys. Lett. B **299**, 234 (1993); A. Brignole, L.E.Ibáñez, C. Muñoz, Nucl. Phys. B **422**, 125 (1994); T. Kobayashi, D. Suematsu, K. Yamada, Y. Yamagishi, Phys. Lett. B **348**, 402 (1995)
12. E. Cremmer, S. Ferrara, C. Kounnas, D.V. Nanopoulos, Phys. Lett. B **133**, 61 (1983); J.R. Ellis, A.B. Lahanas, D.V. Nanopoulos, K. Tamvakis, Phys. Lett. B **134**, 429 (1984); J.R. Ellis, C. Kounnas, D.V. Nanopoulos, Nucl. Phys. B **247**, 373 (1984); A.B. Lahanas, D.V. Nanopoulos, Phys. Rept. **145**, 1 (1987)
13. Z. Chacko, M.A. Luty, JHEP **0105**, 067 (2001); D. Marti, A. Pomarol, Phys. Rev. D **64**, 105025 (2001)
14. T. Kobayashi, K. Yoshioka, Phys. Rev. Lett. **85**, 5527 (2000)
15. G.F. Giudice, R. Rattazzi, Nucl. Phys. B **511**, 25 (1998)

16. J. Scherk, J.H. Schwarz, Phys. Lett. B **82**, 60 (1979); Nucl. Phys. B **153**, 61 (1979)
17. G. von Gersdorff, M. Quiros, Phys. Rev. D **65**, 064016 (2002)
18. I. Antoniadis, S. Dimopoulos, A. Pomarol, M. Quiros, Nucl. Phys. B **544**, 503 (1999); R. Barbieli, L. Hall, Y. Nomura, Phys. Rev. D **63**, 105007 (2001); N. Arkani-Hamed, L.J. Hall, Y. Nomura, D.R. Smith, Nucl. Phys. B **605**, 81 (2001)
19. T. Kobayashi, H. Terao, Prog. Theor. Phys. **107**, 785 (2002)
20. A. Falkowski, C. Grojean, S. Pokorski, Phys. Lett. B **535**, 258 (2002)
21. Y. Hosotani, Phys. Lett. B **126**, 309 (1983); Ann. Phys. **190**, 233 (1989)